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Convective heat transfer and friction factor correlations of nanofluid in a tube and with inserts: A review

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ABSTRACT

In the heat transfer area researches have been carried out over several years for the development of convective heat transfer enhancement techniques. The use of additives in the base fluid like water or ethylene glycol is one of the techniques applied to augment the heat transfer. Recently an innovative nanometer sized particles have been dispersed in the base fluid in heat transfer fluids. The fluids containing the solid nanometer size particle dispersion are called 'nanofluids'. The dispersed solid metallic or nonmetallic nanoparticles change the thermal properties like thermal conductivity, viscosity, specific heat, density, heat transfer and friction factor of the base fluid. Nanofluids are having high thermal conductivity and high heat transfer coefficient compared to single phase fluids. The enhancement in heat transfer coefficient with the effect of Brownian motion of the nanoparticles present in the base fluid. In this paper, a comprehensive literature on the correlations developed for heat transfer and friction factor for different kinds of nanofluids flowing in a plain tube under laminar to turbulent flow conditions have been compiled and reviewed. The review was also extended to the correlations developed for the estimation of heat transfer coefficient and friction factor of nanofluid in a plain tube with inserts under laminar to turbulent flow conditions. However, the conventional correlations for nanofluid heat transfer and friction factor are not suitable and hence various correlations have been developed for the estimation of Nusselt number and friction factor for both laminar and turbulent flow conditions inside a tube with inserts.

Contents

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1. Introduction

Thermal loads are increasing in a wide variety of applications like microelectronics, transportation, lighting, utilization of solar energy for power generation etc. The thermal load control technologies with extended surfaces such as fins and micro-channels have already reached their limits. Hence, the management of high thermal loads in high heat flux applications offers challenges and the thermal conductivity of heat transfer fluid have become vital. Traditional heat transfer fluids like water, engine oil, ethylene glycol, propylene glycol are inherently limited heat transfer capability. To overcome the limited heat transfer capabilities of these traditional fluids, micro/millimeter sized particles of high thermal conductivity suspended in them were considered by Ahuja [1]. The major disadvantage is settlement of these course grained particles in the base fluid. To overcome the problem of particle sedimentation, Choi [2] and his team developed nanometer sized particles. Choi et al. [3] observed 160% thermal conductivity enhancement with carbon nanotubes dispersed in engine oil. The similar trend is also observed by Lee et al. [4], Wang et al. [5], Eastman et al. [6,7]. Das et al. [8] have presented temperature dependent thermal conductivity of nanofluid. Sundar and Sharma [9] have observed 6.52% with Al_2O_3 nanofluid, 24.6% with CuO nanofluid thermal conductivity enhancement at 0.8% compared to water. Naik and Sundar [10] have also observed thermal conductivity enhancement with CuO nanoparticles dispersed into glycol and water mixture. Thermal conductivity of some commonly used solids and liquids as shown in Table 1.

Researchers have investigated the convective heat transfer for single-phase fluids and also developed correlations for the estimation of Nusselt number and friction factor. Instead of using single-phase fluids in heat exchangers, now researchers are investigating the convective heat transfer and feasibility of usage of nanofluids in a device. Nanofluid consists of nanosized particle dispersed in a fluids is called 'nanofluid'. Experimental investigation of convective heat transfer of different kind of nanofluids in a tube has been estimated by many researchers. Xuan and Li [11] have experimentally obtained heat transfer enhancement of Cu/water nanofluid in a tube under laminar flow condition and also developed correlation for Nusselt number.

Wen and Ding [12] experimentally obtained 47% heat transfer enhancement with Al_2O_3 nanofluid at 1.6% volume concentration under the Reynolds number of 1600. Experiments with Al_2O_3/water nanofluid in the laminar flow range of $Re=700$ and 2050 has been conducted by Heris et al. [13] and observed heat transfer augmentation with increase in Peclet number and nanoparticle volume fraction. Ding et al. [14] observed 350% heat transfer enhancement with carbon nanotubes (CNT's) flowing in a horizontal tube at 0.5% weight concentration at Reynolds number is 800. Ho et al. [15] have experimentally investigated the convective heat transfer enhancement in Al_2O_3/water nanofluid in micro-channel for a laminar flow.

Experimental convective heat transfer investigations of Al_2O_3, TiO_2 nanofluids in plain tube under turbulent flow condition are undertaken by Pak and Cho [16] and also developed correlation for Nusselt number. Fotukian and Esfahany [17,18] have observed 25% heat transfer enhancement of Al_2O_3/water and 20% pressure drop enhancement. Duangthongsuk and Wongwises [19] performed experimental studies on 0.2% TiO_2 nanofluid in double tube counter flow heat exchanger and obtained 6–11% heat transfer enhancement. Sundar et al. [20] have numerically obtained 2.25% heat transfer enhancement and 1.42% friction factor for Al_2O_3 nanofluid in a tube. Sundar et al. [21] have estimated the magnetic Fe_3O_4 nanofluid heat transfer in a tube and also presented Nusselt number and friction factor correlations.

Nanofluid is having the following advantages compared to single phase fluid: (i) high dispersion stability with predominant Brownian motion of particles (ii) reduced particle clogging as compared to convention slurries, thus promoting system miniaturization (iii) reduced pumping power as compared to pure liquid to achieve equivalent heat transfer intensification (iv) adjustable properties, including thermal conductivity and surface wettability, by varying particle concentrations to suit different applications (v) high specific surface area and therefore more heat transfer surface between particles and fluids. The enhancement in heat transfer of nanofluid cause several reasons such as Brownian motion, Brownian diffusion, friction factor between the fluid layer and the nanoparticle. It also causes dispersion, layering at the liquid/solid interface, ballistic phonon transport and thermophoresis of the nanofluid. Heat transfer experiments are indicating that thermal conductivity is not only the reason for heat transfer augmentation of the nanofluid; it also depends on the Prandtl number. Proper detailed physical mechanism for nanofluid heat transfer augmentation has not been established.

Experimental heat transfer and friction factor of nanofluid in a tube with different kind of inserts is the interesting topic. Researchers are investigating the further heat transfer enhancement for nanofluid flowing in a tube with different kind of inserts. Chandrasekar et al. [22,23] investigated the heat transfer of Al_2O_3/water nanofluids in a circular tube with wire coil inserts and found heat transfer enhancement of up to 15.91%. Pathipakka and Sivashanmugam [24] numerically investigated heat transfer of 1.5% volume concentration of Al_2O_3/water nanofluid in a tube with twisted tape inserts of 2.93 twist ratio and found 31.29% enhancement in the heat transfer at $Re=2039$. Sundar and Sharma [25] have obtained 22.0% heat transfer enhancement for water in tube with longitudinal strip inserts of $AR=1$. Sundar and Sharma [26] investigated convective heat transfer and friction factor of Al_2O_3 nanofluid in circular tube fitted with twisted tape inserts. Sharma et al. [27], Sundar and Sharma [28,29] presented the empirical correlation for the estimation of Nusselt number and friction factor of Al_2O_3 nanofluid flowing in a tube with twisted and longitudinal strip inserts. Sundar et al. [30] have investigated Fe_3O_4/water nanofluid in a tube with twisted tape inserts and also developed Nusselt number and friction factor correlations.

Convective heat transfer and friction factor of nanofluid flowing in a tube and with different kind of inserts have been explained by

Nomenclature

C_p	specific heat
d	diameter, m
D	inner diameter, m
f	friction factor
H	twisted tape pitch, m
k	thermal conductivity, W/m K
L	tube length, m
Nu	Nusselt number, $h \times D/k$
p	helical pitch, m
Pe	Peclet number, $Pe = u_m dp/\alpha$
Pr	Prandtl number, $\mu \times C_p/k$
Re	Reynolds number, $4 \times m/\pi D\mu$.
x	tube entrance length, m
α	thermal diffusivity, m^2/s
μ	dynamic viscosity, kg/*m s*
ϕ	particle volume concentration
ρ	fluid density, kg/m^3

Subscripts

b	bulk
bf	base fluid
nf	nanofluid
p	nanoparticle

the researchers. They obtained further heat transfer and friction factor enhancement with the use of inserts in a tube. Eqs. (1) and (2) is the general form of the Nusselt number and friction factor of nanofluid flowing in a plain tube given by Xuan and Roetzel [31].

$$Nu_{nf} = f\left(\mathrm{Re}, \mathrm{Pr}, \frac{k_p}{k_{kf}}, \frac{(\rho Cp)_p}{(\rho Cp)_{bf}}, \varphi, \text{ Particle size and shape, flow structure}\right) \tag{1}$$

$$f_{nf} = f(\mathrm{Re}, \varphi, \text{ Particle size and shape, flow structure}) \tag{2}$$

The present study reveals the critical review for the availability of correlations for the estimation of Nusselt number and friction factor of nanofluid flowing in a plain tube with different kind of inserts.

2. Synthesis of nanoparticles

All solid nanoparticles with high thermal conductivity can be used as dispersed material in the base fluid for the preparation of nanofluids. Based on the reported literature the following are the nanoparticles used for nanofluid preparation. (1) carbon nanotube (SWCNT's and MWCNT's) (2) nanodroplet (3) metallic particles (Cu, Al, Fe, Au and Ag) and (4) non-metal particles (Al_2O_3, CuO, Fe_3O_4, TiO_2 and SiC). Thermal conductivity enhancement obtained by various researchers is reported in Table 2. Nanoparticles such as metallic or non-metallic dispersed in the fluids have been widely investigated by many researchers. Recently development with nanodroplets, a new kind of nanofluid was reported Ma et al. [44]. Those fluids are having long term stability and can be easily mass produced. It is doubt with the nanodroplets thermal conductivity enhancement. A nanofluid which contains nanoparticles and liquid metal has been proposed by Zhu et al. [45]. With this the definition of nanofluid needs to be modified. So, the nanofluid is a new kind of composite materials containing nano additives and the base fluid. The additives may be metal or nonmetal nanoparticles, nanofiber, nanorods, nanotubes or nanodroplets and the base fluids are any fluids useful. The investigations on different nanofluid systems are in experimental stage. For engineering applications special nanofluid system is required.

Table 1
Thermal conductivity of some commonly used liquids and solids.

Materials	Thermal conductivity (W/m K)
Engine oil (EO)	0.15
Kerosene	0.15
Ethylene glycol (EG)	0.253
Water	0.613
Titanium dioxide (TiO_2)	8.4
Copper oxide (CuO)	32.9
Alumina (Al_2O_3)	40
Platinum	70
Sodium (Na)	72.3
Iron (Fe)	80
Cadmium (Cd)	92
Graphite	120
Silicon (Si)	148
Aluminum (Al)	237
Aluminum nitride (AlN)	285
Gold (Au)	317
Titanium carbide (TiC)	330
Silicon carbide (SiC)	350
Copper (Cu)	401
Silver (Ag)	429
Carbon nanotube	3000
Diamond	3300

3. Nanofluid preparation

Nanofluid preparation is very important task with the use of nanoparticles for improving the thermal conductivity of base fluids. Two methods are used for producing the nanofluids, (i) single-step method (ii) two-step method. In the single-step method is a process combining the preparation of nanoparticles with the synthesis of nanofluids, for which the nanoparticles are directly prepared by physical vapour deposition (PVD) technique or liquid chemical method. In this method the processes of drying, storage, transportation, and dispersion of nanoparticles are avoided, so the agglomeration of nanoparticles is minimized and the stability of fluids is increased. But the disadvantage of this method is that only low vapour pressure fluids are compatible with the process. Zhu et al. [39] presented a novel single-step chemical method for preparing copper nanofluids by reducing $CuSO4 \cdot 5H_2O$with $NaH_2PO_2 \cdot H_2O$ in ethylene glycol under microwave irradiation and no agglomeration, stability is obtained. Liu et al. [38] synthesized nanofluids containing Cu nanoparticles in water through chemical reduction method for the first time. Eastman and Choi [7] have used a single-step physical method to prepare nanofluids, in which Cu vapour was directly condensed into nanoparticles by contact with a flowing ethylene glycol.

In the two-step method, nanofluid is prepared by dispersing the nanoparticles into the base fluid. Nanoparticles, nanofibers or nanotubes used in this method are first produced as a dry powder by inert gas condensation, chemical vapour deposition, mechanical alloying or other suitable techniques, and the nano-sized powder is then dispersed into a fluid in a second processing step. By the step by step method from preparation of nanoparticle to nanofluid preparation, there is a possibility of agglomeration of the nanoparticles takes place in the base fluid. This agglomeration causes the decrease of

Table 2
Thermal conductivity enhancement of various nanofluid reported in literature.

Author	Nanofluid	Synthesis process	φ (%)	Particle size (mm)	Thermal conductivity, (%)
Patel et al. [32]	Au/Toluene	Two-step	0.00026	10∼20	21 (60 °C)
Patel et al. [32]	Ag/Toluene	Two-step	0.001	60∼80	16.5 (60 °C)
Xie et al. [33]	Al_2O_3/EG	Two-step	0.05	60	29
Liu et al. [34]	Cu/H_2O	Single-step	0.1	75∼100	23.8
Eastman and Choi [7]	Cu/EG	Single-step	0.3	10	40
Hong et al. [35]	Fe/EG	Single-step	0.55	10	18
Putnam et al. [36]	Au/Ethanol	Two-step	0.6	4	1.3 ± 0.8
Xie et al. [37]	CNTs/Decene	Two-step	1.0	15×30 μm	12.7
Xie et al. [37]	CNTs/EG	Two-step	1.0	15×30 μm	19.6
Xie et al. [37]	CNTs/H_2O	Two-step	1.0	15×30 μm	7.0
Choi et al. [3]	CNTs/Poly oil	Two-step	1.0	25×50 μm	160
Liu et al. [38]	CNTs/Engine oil	Two-step	2.0	20∼50 nm	30
Zhu et al. [39]	Fe_3O_4/H_2O	Single-step	4	10	38
Xie et al. [37]	SiC/H_2O	Two-step	4.2	25	15.9
Murshed et al. [40]	TiO_2/H_2O	Two-step	5	15	30–33
Xie et al. [33]	Al_2O_3/H_2O	Two-step	5	20	20
Zhang et al. [41]	CuO/H_2O	Two-step	5	33	11.5
Xuan and Li [42]	Cu/H_2O	Two-step	7.5	100	78
Yang et al. [43]	H_2O/FC-72	Two-step	12	9.8	52

thermal conductivity. Simple techniques such as ultrasonic agitation or adding surfactants to the fluids are used to minimize the particle aggregation. Now, several companies are preparing the nanoparticles by using two-step method, and the nanoparticles are also available in the market. Important thing is, before conducting the experiments with nanofluids make sure that nanoparticles should be uniformly dispersed in the base fluids.

Murshed et al. [40] reported TiO_2 suspension in water prepared by two-step method. Hong et al. [35] prepared Fe nanofluids by dispersing Fe nanocrystalline powder in ethylene glycol by a two-step procedure with a mean diameter of 10 nm and were synthesized by chemical vapour deposition method and used an ultra sonic cell disrupter to avoid the settlement of the nanoparticles. Liu et al. [38] and Choi et al. [3] produced carbon nanotube suspensions by a two-step method. Xie et al. [37] prepared Al_2O_3/H_2O, Al_2O_3/EG, Al_2O_3/PO nanofluids by two-step method, and intensive ultrasonication and magnetic force agitation were employed to avoid nanoparticle aggregation. Xuan and Li [44] prepared Cu/H_2O, Cu/oil nanofluids by two-step method and used ultrasonic agitation to avoid nanoparticle aggregation. Chopkar et al. [46] have synthesized Al_2Cu and Ag_2Al nanoparticles by mechanical alloying and obtained 50–150% thermal conductivity enhancement by dispersing into water and ethylene glycol 0.2–1.5% volume concentration.

3.1. Nanofluid properties

The thermo-physical properties of nanofluid are very important parameters for estimating the heat transfer coefficient. The mixture properties of nanofluids are normally expressed in percentage of volume concentration (ϕ), while the loading analysis was obtained in weight percent (w). The given percentage of volume concentration the weight of nanoparticles required is estimated through Eq. (3). Density and specific heat of nanofluid is estimated using solid–liquid mixture equations. Density and specific heat of the nanofluid is estimated through Eqs. (4) and (5).

$$\varphi\times100=\frac{W\times\rho_f}{\rho_p\ \times(1-W)+W\times\rho_f} \quad (3)$$

$$\rho_{nf}=\varphi\times\rho_p+(1-\varphi)\times\rho_f \quad (4)$$

$$C_{nf}=\frac{\varphi(\rho\times C_p)_p+(1-\varphi)\times(\rho\times C_p)_f}{\rho} \quad (5)$$

The important flow properties like viscosity and thermal conductivity are not only depending on volume concentration of nanoparticle, it also depends on other parameters like particle size, particle shape and surfactant etc. Results from various researchers showed that, the viscosity and thermal conductivity increases with increase of percentage of volume concentration compared to base fluid. Different theoretical and experimental studies have been conducted and various correlations have been proposed for dynamic viscosity and thermal conductivity of nanofluids so far, but generalised correlation is not established.

3.2. Non-dimensional numbers

Nusselt number and friction factor correlations, the following dimensionless parameters like Reynolds number, Prandtl number, Peclet numbers, Nusselt number are introduced:

$$Re=\frac{4\dot{m}}{\pi D\mu} \quad (6)$$

$$Pr=\frac{\mu\,C_p}{k} \quad (7)$$

$$Nu=\frac{h\,D}{k} \quad (8)$$

$$Pe_d=\frac{u_m d_p}{\alpha} \quad (9)$$

where thermal diffusivity of the nanofluid is given by

$$\alpha=\frac{k_{nf}}{(\rho\times C_p)_{nf}}$$

4. Single-phase fluid in a tube

Correlations are available for the estimation of Nusselt number and friction factor for single-phase fluids flowing in a tube under laminar to turbulent flow conditions. Commonly used correlations are given below.

4.1. Nusselt number

(a) Shah [47]

$$Nu=1.953\left(Re\,\mathrm{Pr}\frac{D}{x}\right)^{1/3}\quad\left(Re\,\mathrm{Pr}\frac{D}{x}\right)\geq33.3$$

$$Nu = 4.364 + 0.0722\left(Re\,\mathrm{Pr}\frac{D}{x}\right)\ \left(Re\,\mathrm{Pr}\frac{D}{x}\right) < 33.3 \quad (10)$$

(b) Churchill and Ugasi [48]

$$Nu^{10} = Nu^{10} + \left[exp\left(\frac{2200-(Re/365))}{Nu_{lc}^2} + \frac{1}{Nu_t^2}\right]^{-5} \quad (11)$$

Nu_l=*Laminar Nusselt number*
Nu_{lc}=*Nusselt number at critical Reynolds number of* 2100
Nu_t=*Turbulent Nusselt number*

(c) Tam and Ghajar [49]

$$Nu = 0.0233\ Re^{0.8}\ Pr^{0.385}\left(\frac{x}{D}\right)^{-0.0054}\left(\frac{\mu_b}{\mu_w}\right)^{0.14} \quad (12)$$

$$3 \le \frac{x}{D} \le 192{,}7000 \le Re \le 49{,}000{,}4 \le Pr \le 34{,}1.1 \le \frac{\mu_b}{\mu_w} \le 1.7$$

(d) Sider-Tate [50]

$$Nu = 0.027\ Re^{0.8}\ Pr^{0.3}\left(\frac{\mu}{\mu_s}\right)^{0.14} \quad (13)$$

$$0.7 \le Pr \le 16,\ 700, Re \ge 10,\ 000,\ \frac{L}{d} \ge 10$$

(e) Dittus-Boelter [51]

$$Nu = 0.023\ Re^{0.23} Pr^{0.4} \quad (14)$$

$$0.6 \le Pr \le 160, Re \ge 10,\ 000, \frac{L}{d} \ge 10$$

(f) Gnelinski's [52]

$$Nu = 0.012\ (Re^{0.87} - 280)Pr^{0.4} \quad (15)$$

$$1.5 \le Pr \le 500{,}3000 \le Re \le 10^5$$

Alternate equation for the estimation of Nusselt number is,

$$Nu = \frac{(f/8)(Re-1000)Pr}{1+12.7(f/8)^{1/2}(Pr^{2/3}-1)} \quad (16)$$

$$0.5 \le Pr \le 2 \times 10^3{,}3000 \le Re \le 5 \times 10^6$$

(g) Petukov [53]

$$Nu = \frac{(f/8)(Re-1000)Pr}{1.07+12.7(f/8)^{1/2}(Pr^{2/3}-1)} \quad (17)$$

$$0.5 \le Pr \le 2 \times 10^3{,}10^4 \le Re \le 5 \times 10^6$$

(h) Notter–Sleicher [54]

$$Nu = 5 + 0.015\ Re^{0.856}\ Pr^{0.347} \quad (18)$$

$$0 < Pr < 10^4,\ \ 10^4 < Re < 10^5$$

4.2. Friction factor

(a) Moody [55]

$$f = 0.046\ Re^{-0.20} \quad (19)$$

$$2 \times 10^4 < Re < 10^6$$

(b) Blasius [56]

$$f = 0.079\ Re^{-0.25} \quad (20)$$

$$3 \times 10^3 < Re < 2 \times 10^4$$

(c) Petukov [53]

$$f = (0.790\ ln(Re) - 1.64)^{-2} \quad (21)$$

$$3000 < Re < 5 \times 10^6$$

(d) Filonenko [57] for smooth tubes

$$f = 0.25(0.790\ ln(Re) - 1.64)^{-2} \quad (22)$$

$$3000 < Re < 5 \times 10^6$$

5. Heat transfer coefficient of nanofluid in a tube

5.1. Laminar flow

Experimental and numerical heat transfer of different kinds of nanofluid in a tube has been investigated by many researchers under laminar to turbulent flow conditions and also developed correlations. Some of the nanofluid correlations are given below:

Heris et al. [58] have investigated both Al_2O_3 and CuO nanofluid in a tube under laminar flow. They obtained maximum heat transfer enhancement of 1.29% with CuO and 1.23% with Al_2O_3 nanofluid at 2.5% volume concentration under the Peclet number of 5000. Akbarinia and Behzadmehr [59] have been numerically obtained heat transfer enhancement with Al_2O_3 nanofluid in a horizontal curved tube under fully developed laminar flow condition. Chen et al. [60] have experimentally investigated the titanate nanotubes dispersed in water to form stable nanofluid and they found 13.5% enhancement with 2.5% weight concentration under laminar flow. He et al. [61] have obtained very good heat transfer enhancement with TiO_2 nanofluid in a straight tube under laminar flow conditions by numerically and experimentally.

Hwang et al. [62] have found 8.0% heat transfer enhancement with Al_2O_3 nanofluid at 0.3 wt% under laminar flow. Amrollahi [63] have estimated the convective heat transfer of MWCNT/water nanofluid and found 33–40% enhancement for 0.25% wt. under laminar to turbulent flow. Lajvardi et al. [64] have obtained heat transfer enhancement with ferrofluid magnetic field effect laminar flow conditions and also observed with increase of volume concentration. Bajestan et al. [65] have been numerically obtained heat transfer and pressure drop enhancement with 0.6% of Al_2O_3, CuO, CNT's, and TNT's nanofluids flows through a straight circular pipe in a laminar flow. Huminic and Huminic [66] have numerically found 14% heat transfer enhancement 2.0% volume concentration of CuO nanofluid in double-tube helical heat exchangers under laminar flow.

Anoop et al. [67] have investigated the effect of particle size on the convective heat transfer in Al_2O_3 nanofluid in the developing region and also proposed correlation. It was found nanofluid of 45 nm particles is 25.0% and 150 nm particles shows 11.0% at 4.0% wt.

$$Nu = 4.36 + \left[a \times x_+^{-b}(1+\varphi^c)exp^{(-dx_+)}\right]\left[1+e\left(\frac{d_p}{d_{ref}}\right)^{-f}\right] \quad (23)$$

$a=6.219\times10^{-3},\ b=1.1522,\ c=0.1533,\ d=2.5228,\ e=0.57825$
$f=0.2183,\ d_{ref}=100\ \text{nm},\ d_p=\text{diamter of the particle},\ x_+=\frac{x}{RePrD}$
$50<\frac{x}{D}<200{,}500<Re<2000{,}0\le\varphi\le4\%$

Li and Xuan [68] have found 60% heat transfer enhancement with 2.0% volume concentration of Cu nanofluid under laminar flow and also presented the Nusselt number correlation.

$$Nu=0.4328\ (1+11.258\ \varphi^{0.754}Pe_d^{0.218})Re^{0.333}Pr^{0.4} \quad (24)$$

$800<Re<4000,\ 0<\varphi<2\%$

Yang et al. [69] used graphite nanoparticles in commercial automatic transmission fluid and mixture of synthetic base oils for the preparation of graphite based nanofluid and found at $Re=120$, heat transfer enhancement of 22.0% for 2.5% weight concentration.

$$Nu=a\ Re^bPr^{1/3}\left(\frac{D}{L}\right)^{1/3}\left(\frac{\mu_w}{\mu_b}\right)^{1/3} \quad (25)$$

$5<Re<120{,}0<\varphi<2.5\%$

where ‘a’ and ‘b’ are dependent on nanofluid composition and temperature.

Suresh et al. [70] experimentally investigated the 0.1% volume concentration of Al_2O_3–Cu/water hybrid nanofluids and found 13.56% enhancement in heat transfer at Reynolds number of 1730 compared to water.

$$Nu=0.031(Re\ Pr)^{0.68}(1+\varphi)^{95.73} \quad (26)$$

$Re<2300,\ 0<\varphi<0.1\%$

Rea et al. [71] have observed 27% with alumina/water nanofluid at 6.0% volume concentration and 3.0% with zirconia/water nanofluid at 1.32% volume concentration at fully developed laminar flow condition by considering vertically heated tube.

$$Nu=1.619\ (x^+)^{1/3} \quad (27)$$

$x^+<0.01,\ x^+=\frac{2(x/D)}{Re\ Pr}$
$431<Re<2000,\ 0<\varphi<6.0\%$ for Al_2O_3 nanofluid
$140<Re<362,\ 0<\varphi<3.0\%$ for ZrO_2 nanofluid

5.2. Turbulent flow

The efficiency of solar flat plate collector with multi-walled carbon nanotubes (MWCNT’s) nanofluid have been estimated by Yousefi et al. [72] and they studied upto 0.4% volume concentration. The usage of MWCNT’s based nanofluid in shell and tube exchanger have been investigated by Lotfi et al. [73] and observed heat transfer enhancement compared to base fluid. The effects of the external magnetic field strength and its orientation on the thermal behaviours of the magnetic fluids are analyzed by Li and Xuan [74]. The efficiency of solar flat plate is enhanced to 28.3% by using Al_2O_3 nanofluid at 0.4% wt has been investigated by Yousefi et al. [75].

Zamzamian et al. [76] investigated forced convective heat transfer Al_2O_3/EG and CuO/EG in a double pipe and plate heat exchangers under turbulent flow. The findings indicate considerable enhancement in convective heat transfer coefficient of the nanofluids as compared to the base fluid, ranging from 2% to 50%. Hojjat et al. [77] investigated with Al_2O_3, CuO, and TiO_2 nanoparticles in an aqueous solution of carboxy methyl cellulose (CMC). Peyghambarzadeh et al. [78] have observed 40% heat transfer enhancement with glycol based Al_2O_3 nanofluid in a car radiator compared to base fluid. The heat enhancement of 8.0% for TiO_2 nanofluid at 8.0% volume concentration at $Re=11{,}800$ has been observed by Kayhani et al. [79]. Yu et al. [80] have been observed 50–60% heat transfer enhancement with silicon carbide based nanofluid of 3.7% volume concentration at 3000 to 13000 Reynolds number. Demir et al. [81] have observed heat transfer enhancement for TiO_2/water and Al_2O_3/water nanofluid numerically in a double-tube counters flow heat exchanger.

Ferrouillat et al. [82] have observed 60% heat transfer enhancement with SiO_2/water nanofluids. Bianco et al. [83] have numerically observed heat transfer enhancement with water/Al_2O_3 nanofluid in a circular tube. Namburu et al. [84] considered three nanoparticles like CuO, Al_2O_3 and SiO_2 in an ethylene glycol and water mixture flowing through a circular tube under constant heat flux condition and found 35% enhancement at 6.0% of CuO nanofluid over to other nanofluids. Meibodi et al. [85] considered Al_2O_3 nanofluids for heat transfer estimations. The results show that velocity profile of a nanofluid is similar to the velocity profile of its base fluid. Timofeeva et al. [86] have estimated the heat transfer of SiO_2/TH66 nanofluid under laminar and turbulent conditions and found better performance compared to base fluid.

The effects of Peclet number, volume concentration and particle type on heat characteristics were investigated by Farajollahi et al. [87] considering Al_2O_3/water and TiO_2/water nanofluids in a shell and tube heat exchanger under turbulent flow condition. Heat transfer enhancement for CuO nanofluifd in a helical tube has been analyzed by Hashemi and Behabadi [88] comparing to straight horizontal tube.

Turbulent convective heat transfer for Al_2O_3 and TiO_2 nanofluid in a tube has been analyzed by Pak and Cho [16] experimentally and developed Nusselt number correlation.

$$Nu=0.021Re^{0.8}Pr^{0.5} \quad (28)$$

$10^4<Re<10^5,\ 6.54<Pr<12.33,\ 0<\varphi<3\%$

Turbulent convective heat transfer for Cu nanofluid in a tube has been estimated by Xuan and Li [11] experimentally and also proposed Nusselt number correlation.

$$Nu=0.0059\ (1.0+7.6286\ \varphi^{0.6886}Pe_d^{0.001})Re^{0.9238}Pr^{0.4} \quad (29)$$

$1\times10^4<Re<2.5\times10^4,\ 0<\varphi<2\%$

Duangthongsuk and Wongwises [19] have observed 26.0% heat transfer enhancement for 1.0% of TiO_2 nanofluid and observed 14.0% smaller heat transfer for 2.0% of TiO_2 nanofluid compared to water under same flow condition.

$$Nu=0.074\ Re^{0.707}Pr^{0.385}\varphi^{0.074} \quad (30)$$

$3000<Re<18000,\ 0<\varphi<2\%$

Maïga et al. [89] numerically investigated the laminar forced convection heat transfer behavior of water/Al_2O_3 and ethylene glycol/Al_2O_3 nanofluids in uniformly heated tube. Their study clearly showed that the inclusion of nanoparticles into the base fluids has produced a considerable augmentation of the heat transfer coefficient that clearly increases with an increase of the particle concentration.

$$Nu=0.086\ Re^{0.55}Pr^{0.5}\ \text{constant wall heat flux} \quad (31)$$

$$Nu=0.28\ Re^{0.35}Pr^{0.36}\ \text{constant wall temperature} \quad (32)$$

$Re\le1000,\ 6\le Pr\le753,\ 0<\varphi<10\%$

Maiga et al. [90] proposed correlation for Nusselt number of the water/Al_2O_3 and ethylene glycol/Al_2O_3 mixtures under turbulent flow.

$$Nu=0.085\ Re^{0.71}Pr^{0.35} \quad (33)$$

$10^4 \le Re \le 5 \times 10^5,\ 0 < \varphi < 10\%,\ 6.6 \le Pr \le 13.9$

Sajadi and Kazemi [91] have investigated experimentally with TiO_2/water and found 22% heat transfer enhancement and 25% pressure drop enhancement at 0.25% volume concentration under turbulent flow compared to base fluid.

$$Nu = 0.067\ Re^{0.71} Pr^{0.35} + 0.0005\ Re \tag{34}$$

$5000 < Re < 30{,}000,\ 0 < \varphi < 0.25\%$

Sundar et al. [21] proposed Nusselt number correlation for Fe_3O_4 nanofluid in a tube under turbulent flow. They observed 30.96% heat transfer enhancement compared to water.

$$Nu = 0.02172\ Re^{0.8}\ Pr^{0.5} (1.0 + \varphi)^{0.5181} \tag{35}$$

$3000 < Re < 22{,}000,\ 0 < \varphi < 0.6\%,\ 3.72 < Pr < 6.50$

Buongiorno [92] proposed an alternative explanation for the abnormal heat transfer coefficient increment by considering viscosity within the boundary layer.

$$Nu = \frac{(f/8)(Re - 1000)Pr}{1 + \delta_v^+ (f/8)^{1/2} (Pr^{2/3} - 1)} \tag{36}$$

$\delta_v^+ =$ Thickness of laminar sub layer,that is taken as 15.5
$f =$ friction factor correlation for turbulent flow

$5000 < Re < 65{,}000,\ 0 < \varphi < 3.6\%$ for Al_2O_3 nanofluid
$5000 < Re < 65{,}000, 0 < \varphi < 0.9\%$ for ZrO_2 nanofluid

Asirvatham et al. [93] observed heat transfer enhancement of 28.7% for 0.3% and 69.3% for 0.9% of silver nanofluid respectively and also developed Nusselt number correlation.

$$Nu = 0.0023\ Re^{0.8} Pr^{0.3} + (0.617\varphi - 0.135) Re^{(0.445\varphi - 0.37)} Pr^{(1.081\varphi - 1.305)} \tag{37}$$

$900 < Re < 12{,}100,\ 0 < \varphi < 0.9\%$

Vajjha and Das [94] have considered Al_2O_3, CuO, SiO_2 nanoparticles dispersed in 60:40% of ethylene glycol and water by mass concentration and also proposed correlation based on the thermo-physical properties of the nanofluid.

$$Nu = 0.065 (Re^{0.65} - 60.22)(1 + 0.0169\varphi^{0.15}) Pr^{0.542} \tag{38}$$

$R^2 = 0.97,\ 3000 < Re < 16,000$

$0 < \varphi < 0.06\%$ for CuO & SiO_2 nanofluid
$0 < \varphi < 0.1\%$ for Al_2O_3 nanofluid

Suresh et al. [95] observed heat transfer enhancement of 39% with 0.3% of CuO nanofluid in a helically dimpled tube and also presented the correlation.

$$Nu = 0.00105\ Re^{0.984} Pr^{0.4} (1 + \varphi)^{-80.78} \left(1 + \frac{P}{d}\right)^{2.089} \tag{39}$$

$2500 < Re < 6000,\ 0 < \varphi < 0.3\%$

Table 3
Nusselt number correlations reported in the literature for nanofluid in a tube.

Equation	Nanofluid	φ, (%)	'Re' range	Ref.
$Nu = 4.36 + \left[a \times x_+^{-b} (1 + \varphi^c) exp^{(-dx_+)}\right] \left[1 + e\left(\frac{d_p}{d_{ref}}\right)^{-f}\right]$	Al_2O_3	4.0	500 < Re < 2,000	[67]
$Nu = 0.4328 (1 + 11.258\ \varphi^{0.754} Pe_d^{0.218}) Re^{0.333} Pr^{0.4}$	Cu	2.0	800 < Re < 4,000	[68]
$Nu = a\ Re^b Pr^{1/3} \left(\frac{D}{L}\right)^{1/3} \left(\frac{\mu_w}{\mu_b}\right)^{1/3}$	Graphite	2.5	5 < Re < 120	[69]
	Al_2O_3–Cu	0.1	Re < 2,300	[70]
$Nu = 1.619\ (x^+)^{1/3}$, $x^+ < 0.01$, $x^+ = \frac{2(x/D)}{Re\ Pr}$ $Nu = 0.021 Re^{0.8} Pr^{0.5}$	Al_2O_3	6.0	431 < Re < 2,000	[71]
	ZrO_2	3.0	140 < Re < 362	[71]
	Al_2O_3, TiO_2	3.0	$10^4 <$ Re $< 10^5$	[16]
$Nu = 0.0059 (1.0 + 7.6286\ \varphi^{0.6886} Pe_d^{0.001}) Re^{0.9238} Pr^{0.4}$	Cu	2.0	$10^4 <$ Re $< 2.5 \times 10^4$	[11]
$Nu = 0.074\ Re^{0.707} Pr^{0.385} \varphi^{0.074}$	TiO_2	2.0	3000 < Re < 18,000	[19]
$Nu = 0.086\ Re^{0.55} Pr^{0.5}$ Constant heat flux	Al_2O_3	10.0	Re ≤ 1,000	[89]
$Nu = 0.28\ Re^{0.35} Pr^{0.36}$Constant wall temperature	Al_2O_3	10.0	Re ≤ 1,000	[89]
$Nu = 0.085\ Re^{0.71} Pr^{0.35}$	Al_2O_3	10.0	$10^4 \le$ Re $\le 5 \times 10^5$	[90]
$Nu = 0.067\ Re^{0.71} Pr^{0.35} + 0.0005\ Re$	TiO_2	0.25	5000 < Re < 30,000	[91]
$Nu = 0.02172\ Re^{0.8}\ Pr^{0.5} (1.0 + \varphi)^{0.5181}$	Fe_3O_4	0.6	3000 < Re < 22,000	[21]
$Nu = \frac{(f/8)(Re-1000)Pr}{1 + \delta_v^+ (f/8)^{1/2} (Pr^{2/3} - 1)}$	Al_2O_3	3.6	5000 < Re < 65,000	[92]
	ZrO_2	0.9	5000 < Re < 65,000	
$Nu = 0.0023\ Re^{0.8} Pr^{0.3} + (0.617\varphi - 0.135) Re^{(0.445\varphi - 0.37)} Pr^{(1.081\varphi - 1.305)}$	Silver	0.9	900 < Re < 12,100	[93]
$Nu = 0.065 (Re^{0.65} - 60.22)(1 + 0.0169\varphi^{0.15}) Pr^{0.542}$	CuO, SiO_2	0.06	$3,000 < Re < 16,000$	[94]
$Nu = 0.065 (Re^{0.65} - 60.22)(1 + 0.0169\varphi^{0.15}) Pr^{0.542}$	Al_2O_3	0.1	$3,000 < Re < 16,000$	[94]
$Nu = 0.00105\ Re^{0.984} Pr^{0.4} (1 + \varphi)^{-80.78} \left(1 + \frac{P}{d}\right)^{2.089}$	CuO	0.3	2,500 < Re < 6,000	[95]

Table 4
Friction factor correlations reported in the literature for nanofluid in a tube.

Equation	Nanofluid	φ, (%)	'Re' range	Ref.
$f = 26.4\ Re^{-0.8737}\ (1 + \varphi)^{156.23}$	Al_2O_3–Cu	0.1%	Re < 2,300	[70]
$f = 0.1648\ Re^{0.97} (1 + \varphi)^{107.89} \left(1 + \frac{P}{d}\right)^{-4.463}$	CuO	0.3	2,500 < Re < 6,000	[95]
$f = 0.3491\ Re^{-0.25}\ (1.0 + \varphi)^{0.1517}$	Fe_3O_4	0.6	$3,000 < Re < 22,000$	[21]
$f = 0.3164\ Re^{-0.25} \left(\frac{\rho_{nf}}{\rho_{bf}}\right)^{0.707} \left(\frac{\mu_{nf}}{\mu_{bf}}\right)^{0.108}$	Al_2O_3, CuO	0.06	$3000 < Re < 16,000$	[94]
	SiO_2	0.1	3,000 < Re < 16,000	[94]

Nusselt number correlations for nanofluid in a tube under laminar to turbulent flow conditions obtained by various authors are summarized in Table 3.

6. Friction factor of nanofluid in a tube

Addition of nanoparticles into the base fluid causes the heat transfer enhancement, in the similar way it causes the penalty of pressure drop friction factor across the length of the tube. Researchers found increase in friction factor with addition of nanofluids and also developed correlations.

Suresh et al. [71] experimentally investigated the Al_2O_3–Cu hybrid nanofluid in a tube and obtained correlation for the estimation of friction factor.

$$f = 26.4\, Re^{-0.8737}\,(1+\varphi)^{156.23} \tag{40}$$

$Re < 2300,\ 0 < \varphi < 0.1\%$

Suresh et al. [95] have estimated friction factor for CuO nanofluid by considering dimple tube and found 10.0% enhancement compared to plain tube.

$$f = 0.1648\, Re^{0.97}(1+\varphi)^{107.89}\left(1+\frac{P}{d}\right)^{-4.463} \tag{41}$$

$2500 < Re < 6000,\ 0 < \varphi < 0.3\%$

Sundar et al. [21] experimentally estimated enhancement of friction factor in a plain tube with 0.6% volume concentration of Fe_3O_4 nanofluid when compared to water is 1.09 times and 1.10 times for Reynolds number of 3000 and 22, 000, respectively.

$$f = 0.3491\, Re^{-0.25}\,(1.0+\varphi)^{0.1517} \tag{42}$$

$3000 < Re < 22,000,\ 0 < \varphi < 0.6\%$

Vajjha and Das [94] experimentally investigated with Al_2O_3, CuO and SiO_2 nanofluid in a tube under turbulent flow condition. The increase of 10.0% pressure loss for Al_2O_3 nanofluid at a Reynolds number of 6700.

$$f = 0.3164\, Re^{-0.25}\left(\frac{\rho_{nf}}{\rho_{bf}}\right)^{0.707}\left(\frac{\mu_{nf}}{\mu_{bf}}\right)^{0.108} \tag{43}$$

$4000 < Re < 16,000,\ 0 \le \varphi \le 0.06\%,\ 0 \le \varphi \le 0.1\%$

Friction factor correlations for nanofluid in a tube under laminar to turbulent flow conditions obtained by various authors are summarized in Table 4.

7. Heat transfer coefficient of nanofluid in a tube with inserts

Further heat transfer enhancement of nanofluid in a tube with inserts like twisted tape, helical screw, wire coiled, spiral rod, longitudinal strip has been estimated by some researchers and developed correlations.

7.1. Twisted tape inserts

Schematic diagram of full length twisted tape inserts is shown in Fig. 1. Further heat transfer enhancement for Fe_3O_4 nanofluid in a tube with twisted tape inserts have been experimentally investigated by Sundar et al. [30]. They found that 30.96% enhancement for 0.6% of Fe_3O_4 in a plain tube and further enhancement of 18.49% in a plain tube with twisted tape, $H/D=5$ at the Reynolds number is 22,000.

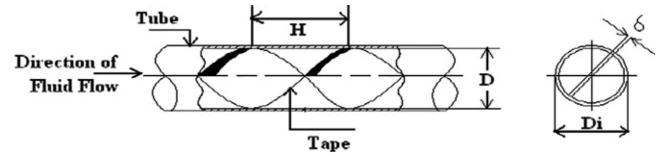

Fig. 1. Full length twisted tape inserts (Sundar et al. [30]).

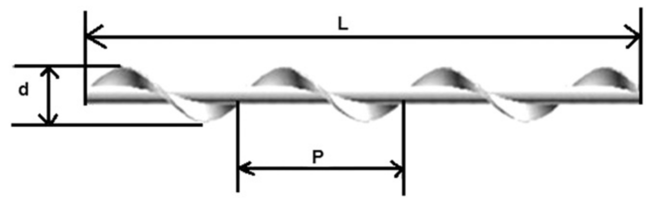

Fig. 2. Schematic diagram of helical screw inserts (Suresh et al. [99]).

$$Nu = 0.0223 Re^{0.8} Pr^{0.5}(1+\varphi)^{0.54}\left(1+\frac{H}{D}\right)^{0.028} \tag{44}$$

$3000 < Re < 22,000,\ 0 < \varphi < 0.6\%,\ 3.19 < Pr < 6.50,\ 0 < \frac{H}{D} < 15$

The heat transfer enhancement of 23.69% for 0.1% of Al_2O_3 nanofluid in a tube has been analyzed by Sharma et al. [27]. Further 44.71% heat transfer enhancement is observed with twisted tape insert with $H/D=5$ inside a circular tube at Reynolds number is 9000.

$$Nu = 3.138\times 10^{-3}(Re)\,(Pr)^{0.6}\,(1+\varphi)^{1.22}\left(1+\frac{H}{D}\right)^{0.03} \tag{45}$$

$3500 < Re < 8500,\ 0 < \varphi < 0.1\%,\ 4.50 < Pr < 5.50,\ 0 < \frac{H}{D} < 15,$

$35 < T_b < 40$

Sundar and Sharma [28] have observed 30.30% heat transfer enhancement for 0.5% of Al_2O_3 in a plain tube, further 42.71% of heat transfer enhancement with twisted tape, $H/D=5$ is observed compared to water at 22,000 Reynolds number.

$$Nu = 0.03666\, Re^{0.8204}\, Pr^{0.4}(0.001+\varphi)^{0.04704}\left(0.001+\frac{H}{D}\right)^{0.06281} \tag{46}$$

$10,000 < Re < 22,000,\ 0 < \varphi < 0.5\%,\ 4.50 < Pr < 5.50,\ 0 < \frac{H}{D} < 83$

Sundar and Sharma [26] laminar convective heat transfer enhancement of 89.76% with 0.5% of Al_2O_3 nanofluid with twisted tape insert of $H/D=5$ compared to water flowing in a plain tube.

$$Nu = 0.5652\, Re^{0.5004}\, Pr^{0.3}(0.001+\varphi)^{0.07060}\left(0.001+\frac{D}{H}\right)^{0.02395} \tag{47}$$

$700 < Re < 2200,\ 0 < \varphi < 0.5\%,\ 4.50 < Pr < 5.50,\ 0 < \frac{H}{D} < 15$

Wongcharee and Eiamsa-ard [96] have observed 1.57 times thermal performance factor for 0.7% of CuO/water nanofluid in a corrugated tube with twisted tape inserts. Wongcharee and Eiamsa-ard [97] have observed Nusselt number increase of 12.8 and 7.2 times with CuO/water nanofluid in a tube with modified twisted tape (TT) and alternative twisted tape inserts (TTAA)

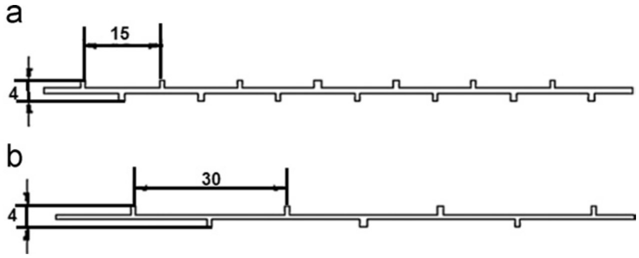

Fig. 3. Schematic diagram of spiral rod inserts (Suresh et al. [100]).

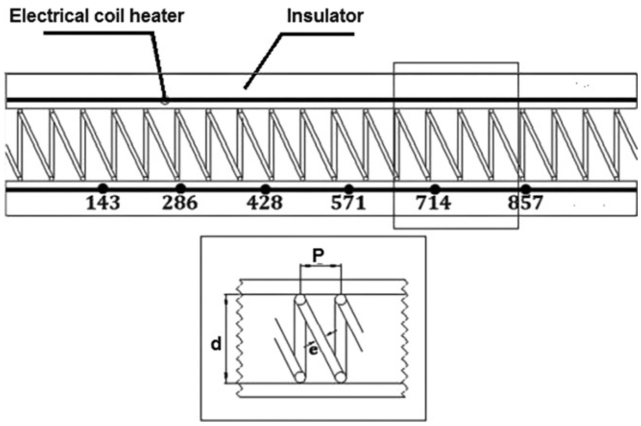

Fig. 4. Schematic diagram of wire coiled inserts (Saeedinia et al. [101]).

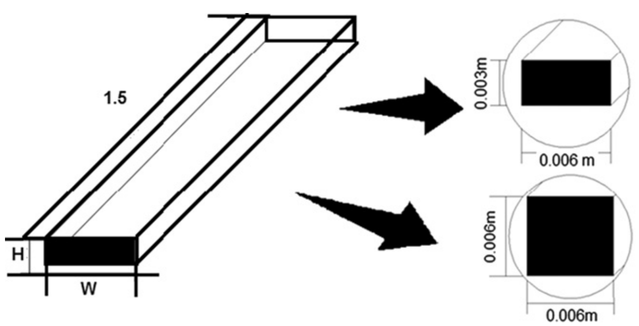

Fig. 5. Schematic diagram of longitudinal strip inserts (Sundar and Sharma [29]).

under laminar flow condition.

$$Nu = 0.026\ Re^{0.927} Pr^{0.4} (1+\varphi)^{0.128} \text{ Twisted tape with alternative axis} \tag{48}$$

$$Nu = 0.005\ Re^{1.062} Pr^{0.4} (1+\varphi)^{0.112} \text{ Twisted tape} \tag{49}$$

$830 < Re < 1990,\ 0 < \varphi < 0.7\%$

7.2. Helical screw tape inserts

Schematic diagram of helical screw tape inserts is shown in Fig. 2. The maximum enhancement of 166.84% for Al_2O_3 nanofluid and 179.82% for CuO nanofluid at twist ratio, $p/d = 1.78$ under the same flow condition with using 0.1% volume concentration have been estimated by Suresh et al. [98] and they conclude that CuO nanofluid is giving better results compared to Al_2O_3 nanofluid.

Comparative study between Al_2O_3 and CuO in a tube with helical tape inserts have been analyzed by Suresh et al. [99] under laminar flow and also proposed correlations.

$$Nu = 0.5419 (Re \times Pr)^{0.53} \left(\frac{P}{d}\right)^{-0.594} \ Al_2O_3 \text{ nanofluid} \tag{50}$$

$$Nu = 0.5657 (Re \times Pr)^{0.5337} \left(\frac{P}{d}\right)^{-0.6062} \text{ CuO nanofluid} \tag{51}$$

$Re < 2300,\ \varphi = 0.1\%$

7.3. Spiral rod inserts

Schematic diagram of spiral rod inserts as shown in Fig. 3. Experimental heat transfer enhancement of 48% for 0.5% of Al_2O_3 nanofluid in a tube with spiral rod inserts have been estimated by Suresh et al. [100].

7.4. Wire coiled inserts

Schematic diagram of wire coiled inserts as shown in Fig. 4. Laminar flow of CuO/base oil nanofluid in a tube with wire coiled inserts have been estimated by Saeedinia et al. [101] experimentally and found 45% enhancement in heat transfer for 0.3% volume concentration and also presented the Nusselt number correlation.

$$Nu = 0.467\ Re^{0.636} Pr^{0.324} \left(\frac{P}{d}\right)^{-0.358} \left(\frac{e}{d}\right)^{0.448} \left(\frac{\mu_s}{\mu_m}\right)^{-0.14} \tag{52}$$

Table 5
Nusselt number correlations reported in the literature for nanofluid in a tube with inserts.

Equation	Nanofluid	φ, (%)	Insert type	‘Re’ range	Ref.
$Nu = 0.0223 Re^{0.8} Pr^{0.5} (1+\varphi)^{0.54} (1+\frac{H}{D})^{0.028}$	Fe_3O_4	0.6	Twisted tape	3000 < Re < 22000	[30]
$Nu = 3.138 \times 10^{-3}\ (Re)\ (Pr)^{0.6} (1+\varphi)^{1.22} (1+\frac{H}{D})^{0.03}$	Al_2O_3	0.1	Twisted tape	3500 < Re < 8500	[27]
$Nu = 0.03666\ Re^{0.8204}\ Pr^{0.4} (0.001+\varphi)^{0.04704} (0.001+\frac{H}{D})^{0.06281}$	Al_2O_3	0.5	Twisted tape	10000 < Re < 22000	[28]
$Nu = 0.5652\ Re^{0.5004}\ Pr^{0.3} (0.001+\varphi)^{0.07060} (0.001+\frac{D}{H})^{0.02395}$	Al_2O_3	0.5	Twisted tape	700 < Re < 2200	[26]
$Nu = 0.026\ Re^{0.927} Pr^{0.4} (1+\varphi)^{0.128}$(TTAA)	CuO	0.7	Twisted tape	830 < Re < 1990	[96]
$Nu = 0.005\ Re^{1.062} Pr^{0.4} (1+\varphi)^{0.112}$(TT)					
$Nu = 0.5419 (Re \times Pr)^{0.53} (\frac{P}{d})^{-0.594}$	Al_2O_3	0.1	Helical tape	Re < 2300	[99]
$Nu = 0.5657 (Re \times Pr)^{0.5337} (\frac{P}{d})^{-0.6062}$	CuO	0.1	Helical tape	Re < 2300	[99]
$Nu = 0.467\ Re^{0.636} Pr^{0.324} (\frac{P}{d})^{-0.358} (\frac{e}{d})^{0.448} (\frac{\mu_s}{\mu_m})^{-0.14}$	CuO	0.3	Wire coiled	10 < Re < 120	[101]
$Nu = 0.279\ (Re \times Pr)^{0.558} (\frac{P}{d})^{-0.477} (1+\varphi)^{134.65}$	Al_2O_3	0.1	Wire coiled	600 < Re < 2275	[22]
$Nu = 0.04532\ Re^{0.7484} Pr^{0.4} (0.001+\varphi)^{0.04373} (0.001+AR)^{0.001} (\frac{D_h}{D_i})^{-0.3345}$	Al_2O_3	0.5	Longitudinal strip	3000 < Re < 22000	[29]

$$10 < Re < 120,\ 0 < \varphi < 0.3\%,\ 0.064 < \frac{e}{d} < 0.107,\ 1.79 < \frac{p}{d} < 2.50$$

Fully developed laminar flow of 0.1% of Al_2O_3 nanofluid in a tube with wire coiled inserts have been analyzed by Chandrasekar et al. [22] have experimentally observed 21.53% heat transfer enhancement with wire coiled insert pitch of 3 and also developed Nusselt number correlation.

$$Nu = 0.279(Re \times Pr)^{0.558}\left(\frac{P}{d}\right)^{-0.477}(1+\varphi)^{134.65} \tag{53}$$

$$600 < Re < 2275,\ 2 \le \frac{P}{d} \le 3,\ 0 < \varphi < 0.1\%$$

7.5. Longitudinal strip inserts

Schematic diagram of longitudinal strip inserts as shown in Fig. 5. Turbulent convective heat transfer of Al_2O_3 nanofluid in a tube with longitudinal strip inserts have been experimentally analyzed by Sundar and Sharma [29]. They found heat transfer enhancement 55.73% with 0.5% of Al_2O_3 nanofluid with longitudinal strip insert of $AR=1$ compared to same fluid in a tube without insert. They also presented the Nusselt number correlation.

$$Nu = 0.04532\,Re^{0.7484}Pr^{0.4}(0.001+\varphi)^{0.04373}(0.001+AR)^{0.001}\left(\frac{D_h}{D_i}\right)^{-0.3345} \tag{54}$$

$$3000 < Re < 22{,}000,\ 0 < \varphi < 0.5\%,\ 4.40 < Pr < 6.20,\ 0 < AR < 18$$

Nusselt number correlations for nanofluid in a tube with different kind of inserts under laminar to turbulent flow conditions obtained by various authors summarized in shown Table 5.

8. Friction factor

With the use of inserts in a flow, causes the enhancement in friction factor also. Researchers estimated friction factor increase with inserts in a flow and also presented correlation.

8.1. Twisted tape inserts

Turbulent friction factor increase of 1.231 times for 0.6% of Fe_3O_4 nanofluid flow in a tube with twisted tape insert of $H/D=5$ has been investigated by Sundar et al. [30] compared to water in a tube and also presented friction factor correlations.

$$f = 0.3490Re^{-0.25}(1+\varphi)^{0.21}\left(1+\frac{H}{D}\right)^{0.017} \tag{55}$$

$$3000 < Re < 22{,}000,\ 0 < \varphi < 0.6\%,\ 3.19 < Pr < 6.50,\ 0 < \frac{H}{D} < 15$$

Transition friction factor of Al_2O_3 nanofluid in a tube with twisted tape inserts presented by Sharma et al. [27] and also developed friction factor correlation.

$$f = 172\,Re^{-0.96}(1+\varphi)^{2.15}\left(1+\frac{H}{D}\right)^{2.15} \tag{56}$$

$$3500 < Re < 8{,}500,\ 0 < \varphi < 0.1\%,\ 4.50 < Pr < 5.50,\ 0 < \frac{H}{D} < 15,$$

$$35 < T_b < 40$$

Turbulent friction factor enhancement of 1.265 times with 0.5% of Al_2O_3 nanofluid in a tube with twisted tape inserts of $H/D=5$ have been analyzed by Sundar and Sharma [28] and also presented friction factor correlation.

$$f = 2.068\,Re^{-0.4330}(1+\varphi)^{0.01}\left(1+\frac{H}{D}\right)^{0.004815} \tag{57}$$

$$10{,}000 < Re < 22{,}000,\ 0 < \varphi < 0.5\%,\ 4.50 < Pr < 5.50,\ 0 < \frac{H}{D} < 83$$

Fully developed laminar friction factor increase of 1.512 times with 0.5% of Al_2O_3 nanofluid in a tube with twisted tape of $H/D=5$ have been observed by Sundar and Sharma [26] and also presented the friction factor correlation.

$$f = 52.08\,Re^{-0.9641}(0.001+\varphi)^{0.01}\left(0.001+\frac{D}{H}\right)^{0.006120} \tag{58}$$

$$700 < Re < 2200,\ 0 < \varphi < 0.5\%,\ 4.50 < Pr < 5.50,\ 0 < \frac{H}{D} < 15$$

Wongcharee and Eiamsa-ard [96] have observed 5.76 times friction factor enhancement for 0.7% of CuO/water nanofluid in a corrugated tube with twisted tape inserts. Wongcharee and Eiamsa-ard [97] have observed friction factor enhancement with CuO/water nanofluid in a tube with modified twisted tape and alternative twisted tape inserts under laminar flow.

$$f = 4.487\,Re^{-0.297}(1+\varphi)^{0.101}\ \text{Twisted tape with alternative axis} \tag{59}$$

$$f = 3.234\,Re^{-0.308}(1+\varphi)^{0.082}\ \text{Twisted tape} \tag{60}$$

$$830 < Re < 1990,\ 0 < \varphi < 0.7\%$$

8.2. Helical screw tape inserts

The friction factor enhancement of 1.22 times for Al_2O_3 and 1.14 times for CuO nanofluid at 0.1% in a tube with helical inserts has been experimentally observed by Suresh et al. [98]. Laminar friction factor of Al_2O_3 and CuO nanofluid in a tube with helical tape insert has been analyzed by Suresh et al. [99] and also proposed separate correlations for Al_2O_3 and CuO nanofluid.

$$f = 177.15\,(Re)^{-0.6821}\left(\frac{P}{d}\right)^{-0.7265}\ Al_2O_3\ \text{nanofluid} \tag{61}$$

$$f = 176.92(Re)^{-0.6811}\left(\frac{P}{d}\right)^{-0.7275}\ \text{CuO nanofluid} \tag{62}$$

$$Re < 2300,\ \varphi = 0.1\%$$

8.3. Spiral rod inserts

Enhancement in friction factor of 8.0% for Al_2O_3 nanofluid in a tube with spiral rod (pitch=30 mm) insert has been analyzed by Suresh et al. [100] under turbulent flow condition.

8.4. Wire coiled inserts

Saeedinia et al. [101] have observed 63.0% pressure drop across the test section by considering CuO/base oil nanofluid in a tube with wire coiled inserts under laminar flow and also proposed correlation.

$$f = 198.7\,Re^{-0.708}\left(\frac{P}{d}\right)^{-0.943}\left(\frac{e}{d}\right)^{0.362}\left(\frac{\mu_s}{\mu_m}\right)^{0.58} \tag{63}$$

$$10 < Re < 120,\ 0 < \varphi < 0.3\%,\ 0.064 < \frac{e}{d} < 0.107,\ 1.79 < \frac{p}{d} < 2.50$$

Table 6
Friction factor correlations reported in the literature for nanofluid in a tube with inserts.

Equation	Nanofluid	φ, (%)	Insert type	Re range	Ref.
$f=0.3490Re^{-0.25}(1+\varphi)^{0.21}\left(1+\frac{H}{D}\right)^{0.017}$	Fe_3O_4	0.6	Twisted tape	3000 < Re < 22000	[30]
$f=172\ Re^{-0.96}(1+\varphi)^{2.15}\left(1+\frac{H}{D}\right)^{2.15}$	Al_2O_3	0.1	Twisted tape	3500 < Re < 8500	[27]
$f=2.068\ Re^{-0.4330}\ (1+\varphi)^{0.01}\left(1+\frac{H}{D}\right)^{0.004815}$	Al_2O_3	0.5	Twisted tape	10000 < Re < 22000	[28]
$f=52.08\ Re^{-0.9641}\ (0.001+\varphi)^{0.01}\left(0.001+\frac{D}{H}\right)^{0.006120}$	Al_2O_3	0.5	Twisted tape	700 < Re < 2200	[26]
$f=4.487\ Re^{-0.297}\left(1+\varphi\right)^{0.101}(\mathrm{TTAA})$	CuO	0.7	Twisted tape	830 < Re < 1990	[96]
$f=3.234\ Re^{-0.308}\left(1+\varphi\right)^{0.082}(\mathrm{TT})$					
$f=177.15(Re)^{-0.6821}\left(\frac{P}{d}\right)^{-0.7265}$	Al_2O_3	0.1	Helical tape	Re < 2300	[99]
$f=176.92(Re)^{-0.6811}\left(\frac{P}{d}\right)^{-0.7275}$	CuO	0.1	Helical tape	Re < 2300	[99]
$f=198.7\ Re^{-0.708}\left(\frac{P}{d}\right)^{-0.943}\left(\frac{e}{d}\right)^{0.362}\left(\frac{\mu_s}{\mu_m}\right)^{0.58}$	CuO	0.3	Wire coiled	10 < Re < 120	[101]
$f=530.8\ Re^{-0.909}\left(\frac{P}{d}\right)^{-1.388}(1+\varphi)^{-512.26}$	Al_2O_3	0.1	Wire coiled	600 < Re < 2275	[22]
$f=1.184\ Re^{-0.3840}\ (0.001+\varphi)^{0.0046}(0.001+AR)^{-0.001}\left(\frac{D_h}{D_i}\right)^{-1.642}$	Al_2O_3	0.5	Longitudinal strip	3000 < Re < 22000	[29]

Chandrasekar et al. [22] have developed friction factor correlation for Al_2O_3 nanofluid in a tube with wire coiled insert.

$$f=530.8\ Re^{-0.909}\left(\frac{P}{d}\right)^{-1.388}(1+\varphi)^{-512.26} \tag{64}$$

$$600<Re<2275,\ 2\leq\frac{P}{d}\leq3,\ 0<\varphi<0.1\%$$

8.5. Longitudinal strip inserts

Friction factor for Al_2O_3 nanofluid flowing in a tube with longitudinal strip inserts under turbulent flow conditions have been analyzed by Sundar et al. [29] and developed correlations.

$$f=1.184\ Re^{-0.3840}(0.001+\varphi)^{0.0046}(0.001+AR)^{-0.001}\left(\frac{D_h}{D_i}\right)^{-1.642} \tag{65}$$

$$3000<Re<22{,}000,\ 0<\varphi<0.5\%,\ 4.40<Pr<6.20,\ 0<AR<18$$

Friction factor correlations for nanofluid in a tube with different kind of inserts under laminar to turbulent flow conditions obtained by various authors are shown in Table 6.

9. Conclusions

The forced convection heat transfer and friction factor correlations for nanofluid in a tube under laminar to turbulent flows conditions are revised. The review also extended to heat transfer and friction factor correlations for nanofluid in a tube with different kind of inserts under laminar to turbulent flow conditions. The review shows that the correlations for Nusselt number and friction factor for both nanofluid in a tube and nanofluid in a tube with inserts have been developed based on both experimental and theoretical studies. Most of the correlations are developed for spherical nanoparticle dispersions.

The single-phase fluid Nusselt number correlations are predicting lower values for nanofluid flowing in a tube. Hence, the conventional correlations are not suitable for estimating the heat transfer coefficient. So, that is the reason for most of the Nusselt number correlations have been suggested for nanofluid in a tube under laminar to turbulent flow conditions. Due to thermophysical properties of nanofluid, particle size, standard mechanism for nanofluid flow causes the large deviation of Nusselt number between the proposed correlations.

Experimental studies related to friction factor of nanofluid is quite matches with the base fluid friction factor correlations. Hence, the fraction factor correlation for single-phase fluid can be use for friction factor prediction of nanofluid.

Further heat transfer enhancement for nanofluid flowing in a tube has been observed with inserts. These inserts create flow obstruction and causes the effective mixing of the fluid within the tube. Geometry of the insert is also an important parameter for heat transfer enhancement. Many experimental data indicating that further heat transfer intensification is possible with inserts. The single-phase and nanofluid correlations are not suitable to predict the Nusselt number for nanofluid in a tube with inserts. Hence, most of the authors developed Nusselt number correlations for nanofluid in a tube with inserts.

The single-phase and nanofluid correlations to predict the friction factor are not suitable for nanofluid in a tube with insert, because with inserts penalty in pressure drop is also high. Hence, separate correlations have been presented for nanofluid in a tube insert.

It is very essential to develop common correlation for nanofluid heat transfer and friction factor in a tube with inserts. Hence further investigations are needed to develop a generalized Nusselt number and friction factor correlations for nanofluid in a tube with inserts.

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